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THEORY OF PULSE DISCHARGES IN A LIQUID MEDIUM

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THEORY OF PULSE DISCHARGES IN A LIQUID MEDIUM * /51

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Study of certain phenomena occurring during underwater spark discharges. Equations for the energy balance, the number of particles, and the rate of expansion of a channel are obtained for underwater discharges during the first quarter of a period. It is shown that there exists a steady-state regime of channel expansion and shock-wave motion, during which time the characteristic parameters have constant values. A calculation is made of these steady-state values.

1. A pulse discharge occurring in a liquid medium is accompanied by the penetration of liquid particles into the channel. The channel is a system with a variable number of particles. This is substantiated by studies on the electric explosion of wires under water (Bibl.1) as well as by the fact that the pressure in the expanding channel remains constant for some time in the presence of an insignificant change in plasma temperature (Bibl.2).

The penetration of particles is conditioned by the heating of the liquid at the periphery of the channel. The heating is chiefly due to collisions between particles of plasma and liquid; the contribution of radiation and recombinations in which third particles take part cannot be significant. The heating results in a gaseous layer between the plasma and the liquid; particles from this layer enter the spark channel where they undergo further heating, dissociation, and partial ionization.

* Numbers in the margin indicate pagination in the original foreign text.

The rate of penetration of particles into the channel is directly proportional to the rate of energy transfer by collisions at the channel periphery and inversely proportional to the vaporization energy per particle. The rate of energy transfer from the i^{th} plasma component may be set at

$$\varepsilon_i' = \frac{N_i u_i \Delta\varepsilon_i}{2a}, \qquad \Delta\varepsilon_i = \frac{4mm_i kT}{(m+m_i)^2} \tag{1.1}$$

Here, N_i is the number of particles of the i^{th} component, u_i is the mean thermal velocity, m and m_i are the masses of the liquid molecule and plasma particle, respectively; a is the channel radius, $\Delta\varepsilon_i$ is the mean energy transferred during collision. From eq.(1.1) and from the gas-kinetic formula $z = 1/4uN/V$, determining the number of collisions between molecules and unit area per unit time, we obtain for the rate of penetration of the particles:

$$N' = \frac{\varkappa\varepsilon'}{q} = 4\left(\frac{2}{\pi}\right)^{1/2} \frac{\varkappa m k^{3/2} N T^{3/2}}{qa} \sum_i \frac{\nu_i m_i^{1/2}}{(m+m_i)^2} \tag{1.2}$$

Here, q is the heat of vaporization per particle.

A theoretical calculation of the coefficient $\varkappa$ is relatively unreliable, since it involves extremely arbitrary assumptions. This coefficient may be determined with the aid of any experimental study serving to plot the curve of the discharge power and to determine an arbitrary channel characteristic. From an analysis of the experimental findings by Skvortsov et al. (Bibl.2), we obtain $\varkappa = 1/24$. /52

2. The energy delivered to the underwater spark channel by the discharge circuit is expended on increasing the internal energy of the channel and on generating a shock wave and radiation; the radiation loss is negligible. An analysis of the current oscillogram and discharge voltages indicates that, during the first quarter of the period, the time dependence of the electric power is

$\mu = m$ i-linear (Bibl.3)

(2.1)

In the presence of low degrees of ionization, the mean energy per plasma particle is

$$\varepsilon = {}^{3}/_{2}kT + \varepsilon_d / \nu$$

where ν is the number of atoms in a liquid molecule, and ε_d is the dissociation energy of the molecule. Then, the change in internal energy of the channel per unit time will be

$$w_i = (N\varepsilon)' = {}^{3}/_{2}kN'T + {}^{3}/_{2}kNT' + N'\varepsilon_d / \nu \tag{2.2}$$

For the power transmitted to the shock wave, the following relation applies:

$$w_g = \frac{NkTV_a'}{V_a} = \frac{2ka'NT}{a} \tag{2.3}$$

Here, a' is the rate of expansion of the channel. Equations of the shock-wave theory indicate that half of this power is expended on compression of the liquid and the other half, on its movement. The relations (1.2), (2.1), (2.2) and (2.3) lead to the equations

$$\begin{aligned} N' &= \frac{\nu\Upsilon}{\varepsilon_d}t - \frac{3k\nu}{2\varepsilon_d}(NT)' - \frac{2k\nu}{\varepsilon_d}\frac{a'}{a}(NT) \\ N'N^{1/2} &= \frac{1}{6}\left(\frac{2}{\pi}\right)^{1/2}\frac{mk^{3/2}}{q}\sum_i \frac{\nu_i m_i^{1/2}}{(m+m_i)^2}\frac{(NT)^{3/2}}{a} \end{aligned} \tag{2.4}$$

The system (2.4) contains three unknown functions (N, NT, a) and must be closed with hydrodynamic equations.

3. The hydrodynamic equations in general form, with constraints for the channel boundary and the shock-wave front cannot be used for solving the problem

because of their high nonlinearity.

A simplifying factor is the experimentally established constancy of the rate of channel expansion during the first quarter of the period (Bibl.2). During its expansion, the discharge channel has the same effect on a liquid as the effect that would be produced by an expanding cylindrical piston. Self-modeling problems of the motion of a medium, displaced by a piston, have been considered in various studies (Bibl.4, 6). In self-modeling problems, the conversion to dimensionless variables transforms the equations of hydrodynamics to a system of ordinary differential equations. However, even at constant rate of piston expansion in water, these equations cannot be integrated in analytic form.

Under such conditions, an additional simplifying factor is required; the incompressibility of the fluid between the channel and the shock front could be adopted as such a factor since, in this region, the fluid is compressed by the shock wave and the subsequent variations in density need not be taken into account in the calculations (Bibl.2).

An integration of the hydrodynamic equations (assuming incompressibility) leads to the equation for the pressure field in the form of /53

$$p = p_a + \frac{\rho_0}{1 - a^2/R^2}\left[\frac{a'^2}{2}\left(1 - \frac{a^2}{r^2}\right) + (aa'' + a'^2)\ln\frac{a}{r}\right] \qquad (3.1)$$

where p_a is the pressure in the channel, ρ_0 is the density of the unperturbed liquid, and R is the radial coordinate of the shock front.

The movement of the channel boundary is directly connected with the propagation of the shock-wave front. The pulse transmitted once every second by the channel to the ambient liquid equals the change in momentum of the liquid between the shock front and the channel. The integral of the pulse for the liquid in this region is cut off by the radial coordinate of the shock front, which eliminates the divergence of the integral.

The expression for the integral of the pulse reads

$$\frac{d}{dt}\int_{a}^{R}\frac{2\pi\rho_0 u r\,dr}{1-a^2/R^2}=2\pi a p_a \tag{3.2}$$

where u is the velocity of the liquid particles. From eq.(3.2), it follows that

$$p_a=\frac{\rho_0}{1-a^2/R^2}\left[(aa''+a'^2)\frac{1-a/R}{a/R}+a'^2\frac{1-a'/D}{a'/D}\right] \tag{3.3}$$

The solution of the self-modeling problem of movement of the medium under the action of a piston expanding at a constant rate leads to the conclusion that the attendant shock wave also propagates at a constant rate and is characterized by a constant pressure at its front. Experimental findings on shock waves generated by underwater discharges confirm the constancy of the wave-front velocity in the presence of a constant rate of channel expansion (Bibl.2). This justifies the assumption that $a'' = 0$, $a/R = a'/D$ in eqs.(3.1) and (3.3) which, after logarithmic expansion, yields

$$p_\varphi=p_a+\frac{\rho_0 a'^2}{2}-\frac{2\rho_0 a'^2}{(1+a'/D)^2},\qquad p_a=\frac{2\rho_0 a' D}{1+a'/D} \tag{3.4}$$

From the first equation in the system (3.4) it follows that, for a channel expanding at a constant rate, the pressure at the shock front is lower than that in the channel. From the Rankine-Hugoniot equations and the equation of continuity of incompressible fluids, we obtain the following formulas:

$$uD=\frac{p_\varphi}{\rho_0},\quad u=\frac{aa'}{R}=\frac{a'^2}{D},\quad \text{or}\quad \frac{p_\varphi}{\rho_0}=a'^2 \tag{3.5}$$

(u = particle velocity at the wave front)

The substitution of eq.(3.5) into eq.(3.4) results in two quadratic equations with respect to the velocity of the front D; a comparison of their coefficients yields the equation for determining the rate of expansion of the channel

$$a'^4 + \frac{p_a}{2\rho_0} a'^2 - \frac{p_a^2}{\rho_0^2} = 0, \qquad \text{or} \qquad a' = 0.8\left(\frac{p_a}{\rho_0}\right)^{1/2}$$

4. The pressure in the channel can be expressed in the form of

$$p_a = \frac{kNT}{\pi a^2 l}$$

From this, taking into account eq.(3.6) and the constancy of the rate of /54 expansion, it follows that

$$NT = \frac{1.6\pi\rho_0 l a'^4 t^2}{k} \tag{4.1}$$

Solving the system (2.4) with respect to NT and equating it with eq.(4.1) we obtain the equation for a'

$$\gamma\left\{5k + 2\frac{\varepsilon_d}{\nu}\left[\frac{1}{12}\left(\frac{2}{\pi}\right)^{1/2}\frac{mk^{1/2}}{qa'}\sum_i \frac{\nu_i m_i^{1/2}}{(m+m_i)^2}\right]^{2/3}\right\}^{-1} = \frac{1.6\pi\rho_0 l a'^4}{k} \tag{4.2}$$

The second term in the denominator on the left-hand side of eq.(4.2) can be neglected; then,

$$a' = \left(\frac{1}{8\pi\rho_0}\gamma_1\right)^{1/4} \qquad \left(\gamma_1 = \frac{\gamma}{l}\right) \tag{4.3}$$

after which the plasma temperature in the channel can be obtained from the solution of eq.(2.4)

$$T = f^{2/3}(\gamma_1)^{1/6}, \qquad f = 6.5q\left(mk^{2/3}\rho_0^{1/4}\sum \frac{\nu_1 m_i^{1/2}}{(m+m_i)^2}\right)^{-1} \tag{4.4}$$

From eq.(4.4) it follows that the plasma temperature remains constant in the presence of a linear increase in the electric power delivered to the channel by the discharge circuit. For the pulse discharges familiar from engineering practice, the ratio γ_1 varies within $3 \times 10^{13} - 3 \times 10^{15}$ w/sec • m at which the temperatures reach $10^4 - 2 \times 10^4$ °K. By reducing the circuit inductance to 0.26 μhenry, Martin (Bibl.1) obtained a value of $\gamma_1 = 1.3 \times 10^{16}$ w/sec • m, which corresponds to a calculated temperature of 2.72×10^4 °K. The channel temperature depends only weakly on γ_1. This explains the fact that the retardation of the discharge on introduction of an inductance is not accompanied by a marked temperature drop.

Solution of the system (2.4), for the number of particles in the channel N, yields

$$N = \frac{(\gamma_1)\, l t^2}{2\varepsilon_d / \nu + 5 k j^{2/3} (\gamma_1)^{1/3}} \tag{4.5}$$

and, for the density of the particle flux at the channel boundary,

$$j_N = \left(\frac{8\rho_0}{\pi^3}\right)^{1/4} \frac{(\gamma_1)^{3/4}}{2\varepsilon_d / \nu + 5 k j^{2/3} (\gamma_1)^{1/3}} \tag{4.6}$$

According to eq.(4.6), the particle flux density in the channel is high, being of the order of $10^{24} - 10^{26}\,\mathrm{sec}^{-1}\,\mathrm{cm}^{-2}$. For the particle density within the channel, eqs.(4.3) and (4.5) yield

$$n = \frac{1.8\, \rho_0^{1/2} (\gamma_1)^{1/2}}{2\varepsilon_d / \nu + 5 k j^{2/3} (\gamma_1)^{1/3}} \tag{4.7}$$

In accordance with eq.(4.7), in the presence of a linear increase in the pulses of the electric power, the density of the particles in the plasma is constant. The decrease in density due to the expansion of the channel is compensated by the influx of particles across the channel wall. The particle density is of the order of $10^{20} - 10^{21}$ cm^{-3} and is more affected by the steepness of γ_1 than by the temperature.

For the plasma pressure, we have the relation /55

$$p_a = \frac{1.8\,\rho_0^{1/4} k f^{1/2} (\gamma_1)^{1/2}}{2\varepsilon_d / \nu + 5 k f^{1/2} (\gamma_1)^{1/6}} \tag{4.8}$$

At γ_1 = const, the channel pressure does not change during the expansion, in view of the constancy of the temperature and of the particle density. Under normal conditions the pressures are of the order of $10^2 - 10^3$ kg/cm^2, but may be higher if the pulse steepness is greater. The pressure depends on γ_1 to a greater extent than on the particle density and to a much greater extent than on the temperature. Hence, low pressures develop in inductance-inhibited discharges at sufficiently high temperatures.

It follows from eqs.(4.3) - (4.8) that, in the presence of a constant steepness of the electric power pulse we are dealing with a steady-state regime of channel expansion, at constant values of temperature, plasma density, plasma pressure, and expansion rate. In this case, a shock front moves at constant velocity and pressure ahead of the expanding channel.

This regime (or one close to it) occurs in underwater sparks from the instant of formation of the discharge channel until the instant of maximum electric power. During this interval of time, the front and near-front regions of the shock wave take form. The steady-state character of the channel expansion results in a trapezoidal shape of the pressure pulses in the shock wave.

5. For a pulse discharge in water, after substitution into eqs.(4.3) - (4.8) of the numerical values of the quantities, we obtain the following working formulas:

$$a' = 7.9 \cdot 10^{-2} \gamma_1^{1/4} \; m/sec., \quad T = 56\, \gamma_1^{1/6}\,{}^\circ\mathrm{K} \tag{5.1}$$

$$N = \frac{\gamma_1 l t^3}{4.3 \cdot 10^{-10} + 3.9 \cdot 10^{-21} \gamma_1^{1/6}} \; \text{particles} \tag{5.2}$$

$$j_N = \frac{4 \gamma_1^{3/4}}{4.3 \cdot 10^{-10} + 3.9 \cdot 10^{-21} \gamma_1^{1/6}} \; sec^{-1} m^{-3} \tag{5.3}$$

$$n = \frac{57\, \gamma_1^{1/2}}{4.3 \cdot 10^{-10} + 3.9 \cdot 10^{-21} \gamma_1^{1/6}} \; m^{-3} \tag{5.4}$$

$$p_a = \frac{4.5 \cdot 10^{-20} \gamma_1^{2/3}}{4.3 \cdot 10^{-10} + 3.9 \cdot 10^{-21} \gamma_1^{2/3}} \quad n/m^2 \qquad (5.5)$$

In these formulas $[\gamma_1] = w/sec \cdot m$.

Table 1 presents the results of calculations for several values of γ_1 as they occur in the pulsed device at $l = 3$ cm.

TABLE 1

γ, w/sec	l, cm	γ_1, w/sec	a', m/sec	T, °K	N particles $t = 5\ \mu sec$	j_N, $sec^{-1}cm^{-3}$	n, cm^{-3}	P_a, kg/cm^2
10^{12}	3	$3.3 \cdot 10^{13}$	188	10 000	$2.2 \cdot 10^{19}$	$5 \cdot 10^{24}$	$2.9 \cdot 10^{20}$	400
$5 \cdot 10^{12}$	3	$1.7 \cdot 10^{14}$	280	13 000	$9.3 \cdot 10^{19}$	$1.4 \cdot 10^{25}$	$5.4 \cdot 10^{20}$	1000
10^{13}	3	$3.3 \cdot 10^{14}$	330	14 600	$1.7 \cdot 10^{20}$	$2.2 \cdot 10^{25}$	$7.0 \cdot 10^{20}$	1500
$5 \cdot 10^{13}$	3	$1.7 \cdot 10^{15}$	500	19 000	$7.2 \cdot 10^{20}$	$5.9 \cdot 10^{25}$	$1.3 \cdot 10^{21}$	3600
10^{14}	3	$3.3 \cdot 10^{15}$	600	21 200	$1.3 \cdot 10^{21}$	$9.0 \cdot 10^{25}$	$1.7 \cdot 10^{21}$	5000

6. Constancy of the velocities and pressures of the shock wave is ensured by the influx of energy from the channel to the shock front across the compressed liquid.

From eqs.(3.5) and (4.3) it follows, for the pressure at the shock front,

$$P_{fr} = {}^1/_8\, \rho_0^{1/3} \gamma_1^{2/3} \qquad (6.1)$$

Substitution of eq.(3.5) into the equation derived by Kirkwood and Bethe /56 (Bibl.7) for shock waves in liquid media

$$D = c_0 + {}^1/_4\, (n + 1)\, u$$

permits obtaining the equation for the velocity of the shock front D.

The solution of this equation reads

$$D = \frac{c_0}{2} \left\{ 1 + \left[1 + \frac{0.4\,(n+1)}{\rho_0^{1/3} c_0^2} \gamma_1^{2/3} \right]^{1/2} \right\}, \qquad c_0 = \left(\frac{\partial p}{\partial \rho} \right)_s^{1/2} \qquad (6.2)$$

Here, c_0 is the speed of sound in the unperturbed medium, and n is an exponent in the equation of state of the medium.

In accordance with eq.(6.2), for a steady-state regime of channel expansion, the shock-wave velocity usually is within 1600 - 2000 m/sec, slightly increasing with increasing γ_1.

After the electric power maximum is reached, the channel characteristics (T, n, p_d) decrease in absolute value together with the energy transmitted to the shock front, thus resulting in a decrease in its velocity and pressure.

The subsequent movement of the front is chiefly determined by the conditions of divergence of the wave energy.

If the channel characteristics and shock front pressure in the regime of steady-state expansion depend only on γ_1, then the wave pressure at a distance from the channel depends not only on γ_1 but also, and to a marked degree, on the rise time of the power pulse τ.

In certain cases, variations in the circuit parameters V, L, C, ℓ may lead to opposite changes in γ_1 and τ; then the wave pressure at a distance from the channel will undergo no substantial changes even if the channel characteristics change markedly in magnitude.

After substitution of the numerical values for water, eqs.(6.1) and (6.2) become

$$p_{fr} = 6.4\,\gamma_1^{1/3}\ n/m^2 \tag{6.3}$$

$$D = 7.5\cdot 10^2\,[1 + (1 + 4.4\cdot 10^{-8}\,\gamma_1^{1/3})^{1/2}]\ m/sec \tag{6.4}$$

7. As a means of converting the electric energy into shock-wave energy, underwater sparks are characterized by an electrohydrodynamic efficiency equal to the ratio of the shock-wave energy to the electric energy delivered to the spark channel by the discharge circuit.

According to eqs.(2.1), (2.3), (4.4), and (4.5), for a steady-state regime of channel expansion at γ_1 = const, the electrohydrodynamic efficiency will have the form

$$\eta_g = \frac{2kf^{2/5}\gamma_1^{1/5}}{2e_d/\nu + 5kf^{2/5}\gamma_1^{1/5}} \tag{7.1}$$

According to eq.(7.1), under normal conditions, η_g is 25 - 30%. In Section 2, we pointed out that half of the energy transmitted to the shock wave is represented by the energy of compression of the liquid and the other half, by the kinetic energy of motion.

For discharges in water, following substitution of the values, eq.(7.1) becomes

$$\eta_g = \frac{1,6\cdot 10^{-21}\gamma_1^{1/5}}{4.3\cdot 10^{-19} + 3.9\cdot 10^{-21}\gamma_1^{1/5}}, \tag{7.2}$$

Equations (7.1) and (7.2) determine η_g only for the ascending segment of /57 the electric power pulse and cannot be used for calculating the efficiency of the discharge as a whole. The conversion of electric energy to hydrodynamic energy takes place continuously so long as power is delivered by the discharge circuit. In addition, part of the energy of the vapor-gas cavity is converted to hydrodynamic energy in the course of post-discharge processes.

Table 2 presents the results of calculations based on eqs.(6.3), (6.4), and (7.2).

TABLE 2

γ_1, w/sec m	P_{fr} kg/cm²	D, m/sec	η_g, %
$3.3\cdot 10^{13}$	364	1600	24.2
$1.7\cdot 10^{14}$	820	1600	26.8
$3.3\cdot 10^{14}$	1150	1750	27.4
$1.7\cdot 10^{15}$	[illegible]	2000	30.2
$3.3\cdot 10^{15}$	[illegible]	2140	30.4

Ioffe et al. (Bibl.8) used a different procedure in deriving a system of equations for the rate of expansion of the channel and its pressure.

8. The degree to which the theoretical findings agree with the experimental data of a series of investigations is shown by Table 3.

TABLE 3

Experimental Study	Circuit Parameter				γ, m/sec	γ₁, m/sec m	Measured Value	Experimental Values	Theoretical Value	No. of Working Formula
	V, kv	C, μf	L, μh	l, cm						
(Bibl. 1)	25	5.8	0.26	1.5	[illegible]	[illegible]	T, °K	20,[illegible]	[illegible]7.200	([illegible].2)
(Bibl. 2)	40	2.7	7	1.5	$1.[illegible] \cdot 10^{13}$	$8.6 \cdot 10^{18}$	a', m/sec	[illegible]	240	([illegible].1)
(Bibl. 2)	40	2.7	7	1.5	$1.3 \cdot 10^{13}$	$8.6 \cdot 10^{18}$	D, m/sec	1600	1630	(6.6)
(Bibl. 8)	6	150	2	7	2.10^{13}	$2.8 \cdot 10^{20}$	a', m/sec	140	180	(5.1)

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